

using four-wave mixing

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Received November 23, 1981

A proposal and an analysis are presented for imaging through a distorting medium using field multiplication by four-wave mixing.

The recent flourish of activity in phase-conjugation optics has revived the interest in real-time imaging through distorting media. Some demonstrations of imaging using thin conventional holograms have been reported earlier¹⁻³ as well as real-time imaging schemes using the basic configuration of phase-conjugation optics.⁴⁻⁶ In this latter case, the image was obtained on the same side of the distortion as the object, which is a disadvantage, if not an impossibility, in many cases of practical interest. An extension of these imaging ideas to the case of real-time nonlinear mixing media has also been considered.⁷

The scheme proposed here is illustrated in Fig. 1. An image $f(\mathbf{x}'')$ is to be transmitted from plane (3) to plane (1) through an intervening distorting medium that is characterized by a transmittance $\exp[-i\Phi(\mathbf{x}')]]$. The proposed method of compensation consists of intersecting in a nonlinear medium (NLM) three waves:

(A) A wave A_1 produced by passing a plane wave through the distortion and then imaging the latter on the NLM.

(B) A picture-bearing wave A_2 arriving at the NLM from the direction opposite to that of A_1 .

(C) A spherical reference wave A_3 counterpropagating along the same axis as the picture-bearing wave A_2 .

While we assume that all three waves propagate along the same axis, there will always be some tolerance for wave A_1 to be slightly off-axis. Our analysis of this situation adopts the following plan: (a) We propagate each of the three fields from their respective starting planes to the NLM. (b) We obtain the nonlinear polarization P_{NL} in the NLM that is proportional to the product of the three fields. (c) We solve for the field at plane (1) that is radiated by P_{NL} . We will show that this field is of the *same form* as that which would have been produced at plane (1) by the propagating picture-bearing wave *without any distortion*.

Using the Fresnel–Kirchhoff diffraction integral and dropping constant phase factors and the time factor $\exp(i\omega t)$ throughout, we write the fields (B) and (C) inside the NLM as:

Picture-wave (B):

$$E_2(\mathbf{x}, z) = \frac{A_2}{\lambda(l_2 - z)} e^{ikz} \int d^2x'' f(\mathbf{x}'') \times \exp \left[-\frac{ik}{2(l_2 - z)} |\mathbf{x} - \mathbf{x}''|^2 \right]. \quad (1)$$

Reference-wave (C):

$$E_3(\mathbf{x}, z) = A_3(z) \exp \left[-ikz - \frac{ik}{2(d_2 + z)} |\mathbf{x}|^2 \right]. \quad (2)$$

Wave (A) is given in plane s, just to the left of the lens,
by

$$E_1(\mathbf{s}) = \frac{A_1}{\lambda d_1} \int d^2\mathbf{x}' \exp \left[-\Phi(\mathbf{x}') - \frac{ik}{2d_1} |\mathbf{x}' - \mathbf{s}|^2 \right]. \quad (3)$$

The effect of the lens is to multiply $E_1(\mathbf{s})$ by $\exp(ik|\mathbf{s}|^2/2f)$. Next we propagate it to the point (\mathbf{x}, z) in the NLM, where it becomes

$$E_1(\mathbf{x}, z) = \frac{A_1}{\lambda^2 d_1 (l_1 + z)} e^{ikz} \int d^2x' \int d^2s \\ \times \exp \left[-i\Phi(\mathbf{x}') - \frac{ik}{2d_1} |\mathbf{x}' - \mathbf{s}|^2 \right] \\ \times \exp \left[\frac{ik}{2f} |\mathbf{s}|^2 - \frac{ik}{2(l_1 + z)} |\mathbf{x} - \mathbf{s}|^2 \right]. \quad (4)$$

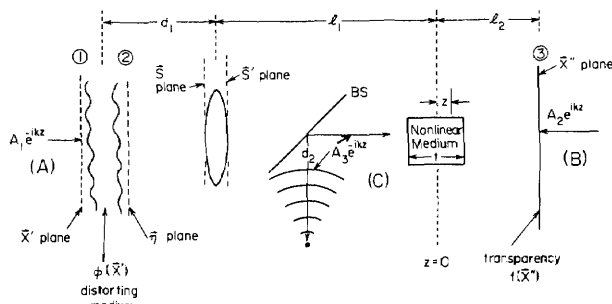


Fig. 1. The configuration for propagating an image from plane (3) through a distorting medium to plane (1) with no distortion.

If the NLM is short, i.e., $z \ll d_2, l_1, l_2$, in the phase factors, we can expand

$$\frac{1}{d_2 + z} \approx \frac{1}{d_2} - \frac{z}{d_2^2}, \quad \frac{1}{l_1 + z} \approx \frac{1}{l_1} - \frac{z}{l_1^2},$$

$$\frac{1}{l_2 - z} \approx \frac{1}{l_2} + \frac{z}{l_2^2} \quad (5)$$

and ignore the z dependence of the denominators preceding the integrals. These relations will be used in what follows.

The intersection of the three waves in the NLM gives rise to a polarization that is proportional to the product of all three. The term of interest is that involving $E_1^* E_2 E_3$ since it oscillates at the fundamental frequency ω and the term E_1^* signifies a reversal of the sign of the distortion. Also this polarization radiates in the desired $-z$ direction toward plane (1). Thus the nonlinear polarization induced in the NLM is given by

$$P_{NL} = \chi^{(3)} E_1^* E_2 E_3, \quad (6)$$

where $\chi^{(3)}$ is the third-order nonlinear susceptibility of the medium. Using the expressions (1), (2), and (4) for the fields as well as (6) and assuming that the distortion is *imaged* by the lens at $z = 0$ so that $f^{-1} = d_1^{-1} + l_1^{-1}$, we obtain

$$P_{NL}(\mathbf{x}, z) = \frac{\chi^{(3)} A_1^* A_2 A_3(z=0)}{\lambda^3 d_1 l_1 l_2} e^{ikz}$$

$$\times \int \int \int d^2 x'' d^2 x' d^2 s f(\mathbf{x}'') e^{i\Phi(\mathbf{x}'')}$$

$$\times \exp \left(-\frac{ik}{2d_2} |\mathbf{x}|^2 - \frac{ikz}{2d_2^2} |\mathbf{x}|^2 \right)$$

$$\times \exp \left(-\frac{ik}{2l_2} |\mathbf{x} - \mathbf{x}''|^2 - \frac{ikz}{2l_2^2} |\mathbf{x} - \mathbf{x}''|^2 \right)$$

$$\times \exp \left[\frac{ik}{2d_1} |\mathbf{x}'|^2 + \frac{ik}{2l_1} |\mathbf{x}|^2 \right.$$

$$\left. - \frac{ik}{d_1} \left(\mathbf{x}' + \frac{d_1}{l_1} \mathbf{x} \right) \cdot \mathbf{s} - \frac{ikz}{2l_1^2} |\mathbf{x} - \mathbf{s}|^2 \right]. \quad (7)$$

The field at the s' plane that is due to P_{NL} is obtained using a Green function solution

$$E(s') = \frac{4\pi^2}{\lambda^2} \int_v \frac{P_{NL}(\mathbf{x}, z) e^{-ikr}}{r} d^2 x dz, \quad (8)$$

where $r = [|\mathbf{x} - \mathbf{s}'|^2 + (l_1 + z)^2]^{1/2}$. In the Fresnel approximation limit this becomes

$$E(s') = \frac{4\pi^2}{\lambda^2 l_1} \int_v d^2 x dz P_{NL}(\mathbf{x}, z)$$

$$\times \exp \left(-ikz - \frac{ik}{2l_1} |\mathbf{x} - \mathbf{s}'|^2 + \frac{ikz}{2l_1^2} |\mathbf{x} - \mathbf{s}'|^2 \right). \quad (9)$$

The effect of the lens again is to multiply $E(s')$ by $\exp(ik|\mathbf{s}'|^2/2f)$. The resulting field propagates to the η plane (2) and is there multiplied by the distortion function $\exp[-i\Phi(\eta)]$. If we recall the imaging condition $f^{-1} = d_1^{-1} + l_1^{-1}$ we obtain after some algebra the desired output field

$$E(\eta) = \frac{4\pi^2 \chi^{(3)} A_1^* A_2 A_3}{\lambda^6 d_1^2 l_1^2 l_2} e^{-i\Phi(\eta)}$$

$$\times \iiint \iiint d^2 x d^2 x' d^2 x'' d^2 s d^2 s'$$

$$\times e^{i\Phi(\mathbf{x}') f(\mathbf{x}'')} \exp \left(-\frac{ik}{2d_1} |\eta|^2 - \frac{ik}{2d_2} |\mathbf{x}|^2 \right.$$

$$\left. + \frac{ik}{2d_1} |\mathbf{x}'|^2 - \frac{ik}{2l_2} |\mathbf{x} - \mathbf{x}''|^2 \right)$$

$$\times \exp \left[-\frac{ik}{d_1} \left(\mathbf{x}' + \frac{d_1}{l_1} \mathbf{x} \right) \cdot \mathbf{s} \right]$$

$$\times \exp \left[\frac{ik}{d_1} \left(\eta + \frac{d_1}{l_1} \mathbf{x} \right) \cdot \mathbf{s}' \right]$$

$$\times \left\{ \exp \left(\frac{ikz}{2d_2^2} |\mathbf{x}|^2 - \frac{ikz}{2l_2^2} |\mathbf{x} - \mathbf{x}''|^2 \right. \right.$$

$$\left. \left. + \frac{ikz}{2l_1^2} |\mathbf{x} - \mathbf{s}'|^2 - \frac{ikz}{2l_1^2} |\mathbf{x} - \mathbf{s}|^2 \right) \right\}. \quad (10)$$

For sufficiently thin NLM such that

$$kz \frac{|\mathbf{x} \text{ or } \mathbf{x}'' \text{ or } \mathbf{s} \text{ or } \mathbf{s}'|^2}{(l_1 \text{ or } l_2 \text{ or } d_2)^2} \ll \pi$$

the last factor in the curly brackets is very nearly unity. Ignoring finite aperture effects, the integrations over \mathbf{s} and \mathbf{s}' yield

$$(d_1 \lambda)^2 \delta \left(\mathbf{x}' + \frac{d_1}{l_1} \mathbf{x} \right) \text{ and } (l_1 \lambda)^2 \delta \left(\mathbf{x} + \frac{l_1}{d_1} \eta \right),$$

respectively. These collapse the \mathbf{x}' and \mathbf{x} integrations to get $\mathbf{x}' \rightarrow \eta$, $\mathbf{x} \rightarrow -(l_1/d_1)\eta$, and then Eq. (10) becomes

$$E(\eta) = [4\pi^2 \chi^{(3)} A_1^* A_3 t / \lambda] \exp \left(-\frac{ik l_1^2}{2d_2 d_1^2} |\eta|^2 \right)$$

$$\times \frac{A_2}{\lambda l_2} \int d^2 x'' f(\mathbf{x}'') \exp \left(-\frac{ik}{2l_2} \left| \frac{l_1}{d_1} \eta + \mathbf{x}'' \right|^2 \right), \quad (11)$$

where t is the thickness (along z) of the NLM.

If $d_2 \rightarrow \infty$, i.e., the reference wave (C) is a plane wave, then we have according to Eq. (11) in the η plane an enlarged (by d_1/l_1) and inverted version of the *original* object field having propagated from an effective plane a distance l_2 to the right *without distortion*. For finite d_2 the resulting field is the same as in Eq. (11) with the additional effect of a lens with focal length $d_2 d_1^2 / l_1^2$ in the η plane. This completes the proof.

The factor in the square brackets preceding the integral in Eq. (11) represents the change in the amplitude of the image field between planes (3) and (1) that is due to nonlinear mixing. This factor is therefore the effective amplitude gain

$$G \approx 4\pi^2 \chi^{(3)} A_1^* A_3 t / \lambda \quad (12)$$

and can exceed unity for sufficiently strong fields. The proposed scheme can thus be used for image amplification as well as correction.

In conclusion, a method for a one-way imaging through a turbulent medium using four-wave mixing in a nonlinear medium has been proposed and analyzed.

The authors would like to acknowledge especially insightful discussions with and valuable suggestions of B. Fischer, J. White, and M. Cronin-Golomb.

This work was supported by the U.S. Army Research Office, Durham, North Carolina.

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